Boolean Outline

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Logic

Logic is the study of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning.¹

Every day, we put logic to work in making decisions about our lives, such as:

- how to dress (e.g., Will it be hot or cold?);
- what to eat and drink (e.g., Will we need caffeine to stay up studying?)
- where to go (e.g., Is it a Monday, in which case I need to go to CS1313?)

We make logical inferences to reason about the decisions we need to make:

- It’s cold this morning, so I need to wear a sweatshirt and jeans, not just a t-shirt and shorts.
- I’ve got a big exam tomorrow that I haven’t studied for, so I’d better drink a couple pots of coffee.
- It’s Monday, so I’d better be on time for CS1313 or I’ll be late for the quiz.

We can even construct more complicated chains of logic:

1. I have a programming project due soon.
2. I have been putting off working on it.
3. Therefore, I must start working on it today.

Symbolic Logic

In logic as in many topics, it sometimes can be easier to manage the various pieces of a task if we represent them symbolically.

- Let $D$ be the statement “I have a programming project due soon.”
- Let $L$ be the statement “I have been putting off working on my programming project.”
- Let $W$ be the statement “I must start working on my programming project today.”

We can then represent the chain of logic like so:

$$D \land L \Rightarrow W$$

This can be read in two ways:

- “$D$ and $L$ implies $W$.”
- “If $D$ is true and $L$ is true, then $W$ is true.”

What if $L$ is not true?

What if I’ve already started working on my programming project? In that case, the statement “I have been putting off working on my programming project” is not true; it is false.

So then the statement

$$D \land L$$

is also false. Why?
Symbolic Logic (continued)

If the statement \( L \) is false, then why is the statement “\( D \) and \( L \)” also false?

Well, in this example, \( L \) is the statement “I have been putting off working on my programming project.” If this statement is false, then the following statement is true: “I haven’t been putting off working on my programming project.”

In that case, the statement \( W \) — “I must start working on my programming project today” — cannot be true, because I’ve already started working on it, so I can’t \textbf{start} working on it now.

\textbf{What if \( D \) is false?}

What if I \textbf{don’t} have a programming project due soon?

Well, statement \( D \) is “I have a programming project due soon.” So if I don’t have a programming project due soon, then statement \( D \) is false.

In that case, statement \( W \) — “I must start working on my programming project today” — is also false, because I don’t have a programming project due soon, so I don’t need to start working on it today.

\textbf{What if both \( D \) and \( L \) are false?}

In that case, I don’t have a programming project due soon, and I’ve already gotten started on the one that’s due in, say, a month, so I definitely don’t need to start working on it today.
Boolean Logic

A Boolean value is a value that is either true or false. The name comes from George Boole, one of the 19th century mathematicians most responsible for formalizing the rules of symbolic logic.

So, in our example, statements D, L and W all are Boolean statements, because each of them is either true or false — that is, the value of each statement is either true or false.

We can express this idea symbolically; for example:

\[
\begin{align*}
D &= \text{true} \\
L &= \text{false} \\
W &= \text{false}
\end{align*}
\]

Note that

\[
L = \text{false}
\]

is read as “The statement L is false,” which in our programming project example means that the statement “I have been putting off working on my programming project” is false, which means that the statement “It is not the case that I have been putting off working on my programming project” is true, which in turn means that the statement “I haven’t been putting off working on my programming project” is true.

So, in this case, “L = false” means that I already have started working on my programming project.
The AND Operation

From this example, we can draw some general conclusions about the statement

“S1 and S2”

for any statement S1 and any statement S2:

- If S1 is true and S2 is true, then “S1 and S2” is true.
- If S1 is false and S2 is true, then “S1 and S2” is false.
- If S1 is true and S2 is false, then “S1 and S2” is false.
- If S1 is false and S2 is false, then “S1 and S2” is false.

We can represent these four sentences in a truth table:

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>AND</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>false</td>
<td>false</td>
<td>false</td>
</tr>
</tbody>
</table>

To read the truth table, put the index finger of your left hand on the value of statement S1 (i.e., either true or false) at the left side of a row, and put the index finger of your right hand on the value of statement S2 at the top of a column. Slide the left index finger rightward, and slide the right index finger downward, until they meet. The value under the two fingers is the value of the statement “S1 and S2.”
Another Boolean Operation

Suppose you want to know whether today is a good day to wear a jacket. You might want to come up with rules to help you make this decision:

- If it’s raining in the morning, then I’ll wear a jacket today.
- If it’s cold in the morning, then I’ll wear a jacket today.

So, for example, if you wake up one morning and it’s cold, then you wear a jacket that day.

We can construct a **general** rule by joining these two rules together:

If it’s raining in the morning

**OR**

it’s cold in the morning,
then I’ll wear a jacket today.

We can apply symbolic logic to this set of statements, like so:

- Let $R$ be the statement “It’s raining in the morning.”
- Let $C$ be the statement “It’s cold in the morning.”
- Let $J$ be the statement “I’ll wear a jacket today.”

We can then represent the chain of logic like so:

$$R \text{ or } C \implies J$$

This can be read in two ways:

- “$R$ or $C$ implies $J$.”
- “If $R$ is true or $C$ is true, then $J$ is true.”
More on OR

What if C is not true? For example, what if it’s hot in the morning? In that case, the statement “It’s cold in the morning” is not true; it is false.

So then what about the statement “R or C”? 

Well, even if it’s hot in the morning, if it’s raining you want your jacket anyway.

In other words, if R is true, then even though C is false, “R or C” is still true.

What if R is false and C is true?

Suppose that it’s not raining in the morning, but it is cold. Then the statement “It’s raining in the morning” is false, and the statement “It’s cold in the morning” is true — and so is the statement “I’ll wear a jacket today.”

In other words, R is false and C is true, and thus “R or C” is also true.

What if both R and C are false?

In that case, it’s neither raining nor cold in the morning, so I won’t wear my jacket.

In other words, if R is false and C is false, then “R or C” is also false.
The OR Operation

From this example, we can draw some general conclusions about the statement 

“S1 or S2”

for any statement S1 and any statement S2:

• If S1 is true and S2 is true, then “S1 or S2” is true.
• If S1 is false and S2 is true, then “S1 or S2” is true.
• If S1 is true and S2 is false, then “S1 or S2” is true.
• If S1 is false and S2 is false, then “S1 or S2” is false.

We can represent these four sentences in a truth table:

<table>
<thead>
<tr>
<th>S1</th>
<th>OR</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>true</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td></td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>

To read the truth table, put the index finger of your left hand on the value of statement S1 (i.e., either true or false) at the left side of a row, and put the index finger of your right hand on the value of statement S2 at the top of a column. Slide the left index finger rightward, and slide the right index finger downward, until they meet. The value under the two fingers is the value of the statement “S1 or S2.”
The NOT Operation

Boolean logic has another very important operation: NOT, which changes a true value to false and a false value to true.

In real life, you’ve probably said something like this:

“I care what you think – NOT!”

Notice that the “NOT” exactly negates the meaning of the sentence: the sentence means “I don’t care what you think.”

From this example, we can draw some conclusions about the statement “not S,” for any statement S:

- If S is true, then “not S” is false.
- If S is false, then “not S” is true.

We can represent these two sentences in a truth table:

<table>
<thead>
<tr>
<th>S</th>
<th>NOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
</tr>
</tbody>
</table>