

Symbolic Logic Outline

1. Symbolic Logic Outline
2. What is Logic?
3. How Do We Use Logic?
4. Logical Inferences #1
5. Logical Inferences #2
6. Symbolic Logic #1
7. Symbolic Logic #2
8. What If a Premise is False? #1
9. What If a Premise is False? #2
10. What If a Premise is False? #3
11. What If Both Premises are False?
12. Boolean Values #1
13. Boolean Values #2
14. Boolean Values #2
15. The AND Operation
16. Truth Table for AND Operation
17. Another Boolean Operation
18. Joining the Premises Together
19. More on OR
20. What If a Premise is False? #1
21. What If a Premise is False? #2
22. What If Both Premises are False?
23. The OR Operation
24. Truth Table for OR Operation
25. Boolean OR is Inclusive
26. What is Exclusive OR?
27. The NOT Operation
28. Truth Table for NOT Operation



What is Logic?

“Logic is the study of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning.”

Irving M. Copi, *Introduction to Logic*, 6th ed., Macmillan Publishing Co., New York, 1982, p. 3.



How Do We Use Logic?

Every day, we put logic to work in making decisions about our lives, such as:

- how to dress

For example:

Will it be hot or cold?

- what to eat and drink

For example:

Will we need caffeine to stay up studying?

- where to go

For example:

Is it a Monday, in which case I need to go to CS1313?



Logical Inferences #1

We make logical *inferences* to reason about the decisions we need to make:

- It's cold this morning, so I need to wear a sweatshirt and jeans, not just a t-shirt and shorts.
- I've got a big exam tomorrow that I haven't studied for, so I'd better drink a couple pots of coffee.
- It's Monday, so I'd better be on time for CS1313 so that I'm on time for the quiz.



Logical Inferences #2

We can even construct more complicated chains of logic:

1. I have a programming project due soon.
2. I have been putting off working on it.
3. Therefore, I must start working on it today.



Symbolic Logic #1

In logic as in many topics, it sometimes can be easier to manage the various pieces of a task if we represent them **symbolically**.

- Let **D** be the statement
“I have a programming project due soon.”
- Let **L** be the statement
“I have been putting off working on my programming project.”
- Let **W** be the statement
“I must start working on my programming project today.”

We can then represent the chain of logic like so:

$$\mathbf{D \text{ and } L \Rightarrow W}$$



Symbolic Logic #2

$$\mathbf{D \text{ and } L \Rightarrow W}$$

This can be read in two ways:

- “**D** and **L** *implies* **W**.”
- “**If** **D** is true **and** **L** is true, **then** **W** is true.”



What If a Premise is False? #1

D and L \Rightarrow W

What if **L** is not true?

What if I've already started working on my programming project?

In that case, the statement

“I have been putting off working on my programming project”
is not true; it is false.

So then the statement

D and L

is also false. Why?



What If a Premise is False? #2

D and $L \Rightarrow W$

If the statement L is false, then why is the statement “ D and L ” also false?

Well, in this example, L is the statement

“I have been putting off working on my programming project.”

If this statement is false, then the following statement is true:

“I haven't been putting off working on my programming project.”

In that case, the statement W –

“I must start working on my programming project today.” –
cannot be true, because I've already started working on it,
so I can't start working on it now.



What If a Premise is False? #3

$$\mathbf{D} \text{ and } \mathbf{L} \Rightarrow \mathbf{W}$$

What if **D** is false?

What if I don't have a programming project due soon?

Well, statement **D** is:

“I have a programming project due soon.”

So if I don't have a programming project due soon,
then statement **D** is false.

In that case, statement **W** –

“I must start working on my programming project today.”
– is also false, because I don't have a programming project
due soon, so I don't need to start working on it today.



What If Both Premises are False?

$$D \text{ and } L \Rightarrow W$$

What if **both** **D** and **L** are **false**?

In that case, I **don't** have a programming project due soon,
and I've **already** gotten started on the one that's due in,
say, a month.

So I definitely **don't** need to start working on it today.

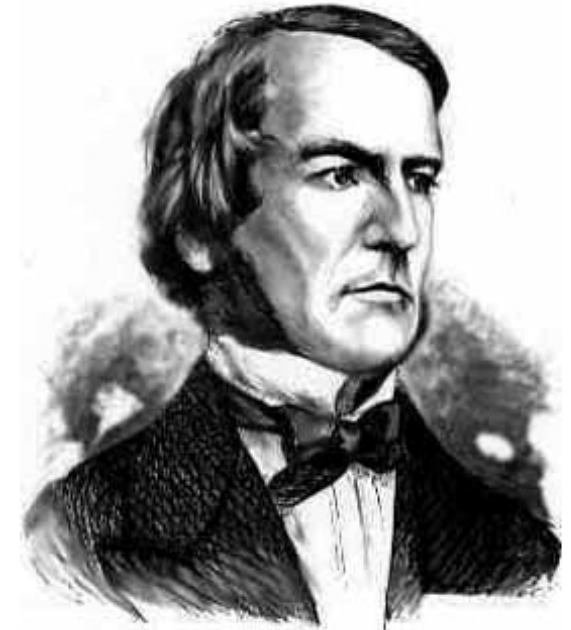


Boolean Values #1

A **Boolean** value is a **value** that is **either true or false**.

The name **Boolean** comes from George Boole, one of the 19th century mathematicians most responsible for formalizing the rules of symbolic logic.

So, in our example, statements **D**, **L** and **W** all are Boolean statements, because each of them is either true or false – that is, the **value** of each statement is either true or false.



<http://thefilter.blogs.com/photos/uncategorized/boole.jpg>



Boolean Values #2

$$\mathbf{D \text{ and } L \Rightarrow W}$$

We can express this idea symbolically; for example:

$$\mathbf{D = true}$$

$$\mathbf{L = false}$$

$$\mathbf{W = false}$$

Note that

$$\mathbf{L = false}$$

is read as “The statement **L** is false.”



Boolean Values #2

$L = \text{false}$

is read as “The statement L is false.”

In our programming project example, this means that the statement

“I have been putting off working on my programming project” is false, which means that the statement

“It is not the case that I have been putting off working on my programming project”

is true, which in turn means that the statement

“I haven't been putting off working on my programming project” is true.

So, in this case, “ $L = \text{false}$ ” means that I already have started working on my programming project.



The AND Operation

From this example, we can draw some general conclusions about the statement

“**S1 and S2**”

for any statement **S1** and any statement **S2**:

- If **S1** is true and **S2** is true, then “**S1 and S2**” is true.
- If **S1** is false and **S2** is true, then “**S1 and S2**” is false.
- If **S1** is true and **S2** is false, then “**S1 and S2**” is false.
- If **S1** is false and **S2** is false, then “**S1 and S2**” is false.



Truth Table for AND Operation

“S1 and S2”

We can represent this statement with a truth table:

		S2	
	AND	true	false
S1	true	true	false
	false	false	false

To read this, put your left index finger on the value of statement **S1** (that is, either true or false) at the left side of a row, and put your right index finger on the value of statement **S2** at the top of a column. Slide your left index finger rightward, and slide your right index finger downward, until they meet. The value under the two fingers is the value of the statement “**S1 and S2.**”



Another Boolean Operation

Suppose you want to know whether today is a good day to wear a jacket. You might want to come up with rules to help you make this decision:

- If it's raining in the morning, then I'll wear a jacket today.
- If it's cold in the morning, then I'll wear a jacket today.

So, for example, if you wake up one morning and it's cold, then you wear a jacket that day.

Likewise, if you wake up one morning and it's raining, then you wear a jacket that day.



Joining the Premises Together

We can construct a **general** rule by joining these two rules together:

If it's raining in the morning

OR

it's cold in the morning,

THEN

I'll wear a jacket today.



More on OR

We can apply symbolic logic to this set of statements, like so:

- Let **R** be the statement
“It’s raining in the morning.”
- Let **C** be the statement
“It’s cold in the morning.”
- Let **J** be the statement
“I’ll wear a jacket today.”

We can then represent the chain of logic like so:

$$\mathbf{R \text{ or } C \Rightarrow J}$$

This can be read in two ways:

- “**R or C implies J.**”
- “**If R is true or C is true, then J is true.**”



What If a Premise is False? #1

What if **C** is not true?

For example, what if it's hot in the morning?

In that case, the statement
“It's cold in the morning”
is not true; it is false.

So then what about the statement “**R or C**”?

Well, even if it's hot in the morning,
if it's raining then you want your jacket anyway.

In other words, if R is true, then even though C is false,
still “R or C” is true.



What If a Premise is False? #2

Suppose that it's not raining in the morning, but it is cold.

Then the statement

“It's raining in the morning.”

is **false**,

and the statement

“It's cold in the morning.”

is **true**.

In which case, the statement

“I'll wear a jacket today.”

is **true**.

In other words, if **R** is **false** and **C** is **true**, then

“**R** or **C**” is also **true**.



What If Both Premises are False?

What if **both** **R** and **C** are **false**?

In that case, it's neither raining nor cold in the morning,
so I won't wear my jacket.

In other words, if **R** is **false** and **C** is **false**,
then “**R or C**” is **false**.



The OR Operation

From this example, we can draw some general conclusions about the statement

“**S1 or S2**”

for **any** statement **S1** and **any** statement **S2**:

- If **S1** is **true** and **S2** is **true**, then “**S1 or S2**” is **true**.
- If **S1** is **false** and **S2** is **true**, then “**S1 or S2**” is **true**.
- If **S1** is **true** and **S2** is **false**, then “**S1 or S2**” is **true**.
- If **S1** is **false** and **S2** is **false**, then “**S1 or S2**” is **false**.



Truth Table for OR Operation

“S1 or S2”

We can represent this statement with a truth table:

		S2	
		OR	
		true	false
S1	true	true	true
	false	true	false

To read this, put your left index finger on the value of statement **S1** (that is, either true or false) at the left side of a row, and put your right index finger on the value of statement **S2** at the top of a column. Slide your left index finger rightward, and slide your right index finger downward, until they meet. The value under the two fingers is the value of the statement “**S1 or S2.**”



Boolean OR is Inclusive

In symbolic logic, the Boolean operation OR is *inclusive*, meaning that it can be the case that both statements are true.

In the jacket example, if it's raining and it's cold, then you'll take your jacket.

So Boolean OR is equivalent to “and/or” in normal colloquial speaking.



What is Exclusive OR?

We know that the Boolean **OR** operation is **inclusive**.

But, there's also such a thing as **exclusive OR**, denoted **XOR**.

XOR is like OR, except that if **both** statements are **true**, then the **result is false**.

We **WON'T** be worrying about XOR in this course.



The NOT Operation

Boolean logic has another very important operation: NOT, which changes a true value to false and a false value to true.

In real life, you've probably said something like this:

“I care what you think – NOT!”

Notice that the NOT exactly negates the meaning of the sentence: the sentence means “I don't care what you think.”

From this example, we can draw some conclusions about the statement “not **S**,” for **any** statement **S**:

- If **S** is true, then “not **S**” is false.
- If **S** is false, then “not **S**” is true.



Truth Table for NOT Operation

“NOT S”

We can represent this statement with a truth table:

	S	
NOT	true	false
	false	true

