Sorting Lesson Outline

1. Sorting Lesson Outline
2. How to Sort an Array?
3. Many Sorting Algorithms
4. Bubble Sort #1
5. Bubble Sort #2
6. Bubble Sort #3
7. Bubble Sort #4
8. Bubble Sort #5
9. Bubble Sort Run Start
10. Bubble Sort Run Iteration #1
11. Bubble Sort Run Iteration #2
12. Bubble Sort Run Iteration #3
13. Bubble Sort Run Iteration #4
14. Bubble Sort Run Iteration #5
15. Bubble Sort Run Iteration #6
16. Bubble Sort Run Iteration #7
17. Bubble Sort Run Iteration #8
18. Bubble Sort Run End
19. Time Complexity of Bubble Sort #1
20. Time Complexity of Bubble Sort #2
21. Selection Sort
22. Selection Sort #1
23. Selection Sort #2
24. Selection Sort Run Start
25. Selection Sort Run Minimum #1
26. Selection Sort Run Minimum #2
27. Selection Sort Run Minimum #3
28. Selection Sort Run Minimum #4
29. Selection Sort Run Minimum #5
30. Selection Sort Run Minimum #6
31. Selection Sort Run Minimum #7
32. Selection Sort Run Minimum #8
33. Selection Sort Run End
34. Time Complexity of Selection Sort #1
35. Time Complexity of Selection Sort #2
36. Quick Sort #1
37. Quick Sort #2
38. Quick Sort #3
39. Quick Sort #4
40. Quick Sort #5
41. Quick Sort Run Start
42. Quick Sort Run Call: low 0, high 8
43. Quick Sort Run Call: low 0, high 3
44. Quick Sort Run Call: low 2, high 3
45. Quick Sort Run Call: low 0, high 3 End
46. Quick Sort Run Call: low 5, high 8
47. Quick Sort Run Call: low 5, high 6
48. Quick Sort Run Call: low 7, high 8
49. Quick Sort Run Call: low 5, high 8 End
50. Quick Sort Run Call: low 0, high 8 End
51. Quick Sort Run End
52. Time Complexity of Quick Sort #1
53. Time Complexity of Quick Sort #2
How to Sort an Array?

| -23 | 97  | 18  | 21  | 5   | -86 | 64  | 0   | -37 |

Suppose you have a big array full of data, and you want to put it into order, either from lowest to highest (ascending) or highest to lowest (descending).

How can you sort it so that it’s in the order you want?
Many Sorting Algorithms

- Bubble Sort
- Selection Sort
- Quick Sort

… and many more.
Bubble Sort #1

```c
#include <stdio.h>

int main ()
{ /* main */
    const int minimum_length = 1;
    const int first_element = 0;
    const int program_failure_code = -1;
    const int program_success_code = 0;
    int* array = (int*)NULL;
    int length, element;
    void bubble_sort(int* array, int length);
    void print_array(int* array, int length);

    http://en.wikipedia.org/wiki/Bubble_sort
```
Bubble Sort #2

```c
printf("I'm going to sort an array for you.\n");
printf("How many elements will the array have?\n");
scanf("%d", &length);
if (length < minimum_length) {
    printf("ERROR: can't have an array of length %d.\n",
            length);
    exit(program_failure_code);
} /* if (length < minimum_length) */
array = (int*)malloc(sizeof(int) * length);
if (array == (int*)NULL) {
    printf("ERROR: can't allocate an array of length %d.\n",
            length);
    exit(program_failure_code);
} /* if (array == (int*)NULL) */
printf("What are the %d elements, unsorted?\n", length);
for (element = first_element; element < length; element++) {
    scanf("%d", &array[element]);
} /* for element */
bubble_sort(array, length);
printf("The %d elements, sorted, are:\n", length);
print_array(array, length);
return program_success_code;
} /* main */
```
void bubble_sort (int* array, int length)
{
    /* bubble_sort */
    const int first_element = 0;
    const int first_iteration = 0;
    const char false = 0;
    const char true = 1;
    int temporary;
    int element;
    int iteration;
    char this_iteration_had_a_swap; /* Boolean */
    void print_array(int* array, int length);
Bubble Sort #4

iteration = first_iteration;
this_iteration_had_a_swap = true;
while (this_iteration_had_a_swap) {
    printf("\n");
    this_iteration_had_a_swap = false;
    for (element = first_element;
         element < (length - 1); element++) {
        if (array[element] > array[element + 1]) {

            /* Swap! */
            temporary = array[element];
            array[element] = array[element + 1];
            array[element + 1] = temporary;

            this_iteration_had_a_swap = true;
        } /* if (array[element] > array[element + 1]) */
    } /* for element */
    printf("After iteration #\%d", iteration);
    printf(" comparing elements #\%d/%d:\n",
            element, element + 1);
    print_array(array, length);
} /* for first_element */
iteration++;
} /* while (this_iteration_had_a_swap) */
} /* bubble_sort */
void print_array (int* array, int length)
{ /* print_array */
    const int first_element = 0;
    int element;

    for (element = first_element;
        element < length; element++) {
        printf("%d ", array[element]);
    } /* for element */
    printf("\n");
} /* print_array */
% bubblesort

I'm going to sort an array for you.
How many elements will the array have?
9
What are the 9 elements, unsorted?
-23  97  18  21   5  -86  64   0  -37
Bubble Sort Run Iteration #1

After iteration #1 comparing elements #0/1:
-23  97  18  21  5  -86  64  0  -37

After iteration #1 comparing elements #1/2:
-23  18  97  21  5  -86  64  0  -37

After iteration #1 comparing elements #2/3:
-23  18  21  97  5  -86  64  0  -37

After iteration #1 comparing elements #3/4:
-23  18  21  5  97  -86  64  0  -37

After iteration #1 comparing elements #4/5:
-23  18  21  5  -86  97  64  0  -37

After iteration #1 comparing elements #5/6:
-23  18  21  5  -86  64  97  0  -37

After iteration #1 comparing elements #6/7:
-23  18  21  5  -86  64  0  97  -37

After iteration #1 comparing elements #7/8:
-23  18  21  5  -86  64  0  -37  97
Bubble Sort Run Iteration #2

After iteration #2 comparing elements #0/1:
-23 18 21 5 -86 64 0 -37 97

After iteration #2 comparing elements #1/2:
-23 18 21 5 -86 64 0 -37 97

After iteration #2 comparing elements #2/3:
-23 18 5 21 -86 64 0 -37 97

After iteration #2 comparing elements #3/4:
-23 18 5 -86 21 64 0 -37 97

After iteration #2 comparing elements #4/5:
-23 18 5 -86 21 64 0 -37 97

After iteration #2 comparing elements #5/6:
-23 18 5 -86 21 0 64 -37 97

After iteration #2 comparing elements #6/7:
-23 18 5 -86 21 0 -37 64 97

After iteration #2 comparing elements #7/8:
-23 18 5 -86 21 0 -37 64 97
After iteration #3 comparing elements #0/1:
-23 18 5 -86 21 0 -37 64 97

After iteration #3 comparing elements #1/2:
-23 5 18 -86 21 0 -37 64 97

After iteration #3 comparing elements #2/3:
-23 5 -86 18 21 0 -37 64 97

After iteration #3 comparing elements #3/4:
-23 5 -86 18 21 0 -37 64 97

After iteration #3 comparing elements #4/5:
-23 5 -86 18 0 21 -37 64 97

After iteration #3 comparing elements #5/6:
-23 5 -86 18 0 -37 21 64 97

After iteration #3 comparing elements #6/7:
-23 5 -86 18 0 -37 21 64 97

After iteration #3 comparing elements #7/8:
-23 5 -86 18 0 -37 21 64 97
Bubble Sort Run Iteration #4

After iteration #4 comparing elements #0/1:
-23 5 -86 18 0 -37 21 64 97

After iteration #4 comparing elements #1/2:
-23 -86 5 18 0 -37 21 64 97

After iteration #4 comparing elements #2/3:
-23 -86 5 18 0 -37 21 64 97

After iteration #4 comparing elements #3/4:
-23 -86 5 0 18 -37 21 64 97

After iteration #4 comparing elements #4/5:
-23 -86 5 0 -37 18 21 64 97

After iteration #4 comparing elements #5/6:
-23 -86 5 0 -37 18 21 64 97

After iteration #4 comparing elements #6/7:
-23 -86 5 0 -37 18 21 64 97

After iteration #4 comparing elements #7/8:
-23 -86 5 0 -37 18 21 64 97
Bubble Sort Run Iteration #5

After iteration #5 comparing elements #0/1:
-86, -23, 5, 0, -37, 18, 21, 64, 97

After iteration #5 comparing elements #1/2:
-86, -23, 5, 0, -37, 18, 21, 64, 97

After iteration #5 comparing elements #2/3:
-86, -23, 0, 5, -37, 18, 21, 64, 97

After iteration #5 comparing elements #3/4:
-86, -23, 0, -37, 5, 18, 21, 64, 97

After iteration #5 comparing elements #4/5:
-86, -23, 0, -37, 5, 18, 21, 64, 97

After iteration #5 comparing elements #5/6:
-86, -23, 0, -37, 5, 18, 21, 64, 97

After iteration #5 comparing elements #6/7:
-86, -23, 0, -37, 5, 18, 21, 64, 97

After iteration #5 comparing elements #7/8:
-86, -23, 0, -37, 5, 18, 21, 64, 97
Bubble Sort Run Iteration #6

After iteration #6 comparing elements #0/1:
-86 -23 0 -37 5 18 21 64 97

After iteration #6 comparing elements #1/2:
-86 -23 0 -37 5 18 21 64 97

After iteration #6 comparing elements #2/3:
-86 -23 -37 0 5 18 21 64 97

After iteration #6 comparing elements #3/4:
-86 -23 -37 0 5 18 21 64 97

After iteration #6 comparing elements #4/5:
-86 -23 -37 0 5 18 21 64 97

After iteration #6 comparing elements #5/6:
-86 -23 -37 0 5 18 21 64 97

After iteration #6 comparing elements #6/7:
-86 -23 -37 0 5 18 21 64 97

After iteration #6 comparing elements #7/8:
-86 -23 -37 0 5 18 21 64 97
Bubble Sort Run Iteration #7

After iteration #7 comparing elements #0/1:
-86 -23 -37 0 5 18 21 64 97

After iteration #7 comparing elements #1/2:
-86 -37 -23 0 5 18 21 64 97

After iteration #7 comparing elements #2/3:
-86 -37 -23 0 5 18 21 64 97

After iteration #7 comparing elements #3/4:
-86 -37 -23 0 5 18 21 64 97

After iteration #7 comparing elements #4/5:
-86 -37 -23 0 5 18 21 64 97

After iteration #7 comparing elements #5/6:
-86 -37 -23 0 5 18 21 64 97

After iteration #7 comparing elements #6/7:
-86 -37 -23 0 5 18 21 64 97

After iteration #7 comparing elements #7/8:
-86 -37 -23 0 5 18 21 64 97
Bubble Sort Run Iteration #8

After iteration #8 comparing elements #0/1:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #1/2:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #2/3:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #3/4:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #4/5:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #5/6:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #6/7:
-86  -37  -23   0   5  18  21  64  97

After iteration #8 comparing elements #7/8:
-86  -37  -23   0   5  18  21  64  97
Bubble Sort Run End

The 9 elements, sorted, are:
-86 -37 -23 0 5 18 21 64 97
Time Complexity of Bubble Sort #1

iteration = firstIteration;
this_iteration_had_a_swap = true;
while (this_iteration_had_a_swap) {
    printf("\n");
    this_iteration_had_a_swap = false;
    for (element = first_element;
         element < (length - 1); element++) {
        if (array[element] > array[element + 1]) {
            /* Swap! */
            temporary = array[element];
            array[element] = array[element + 1];
            array[element + 1] = temporary;
            this_iteration_had_a_swap = true;
        } /* if (array[element] > array[element + 1]) */
    } /* for element */
    printf("After iteration #\d", iteration);  
    printf(" comparing elements #\d/%d:\n", 
            element, element + 1);
    print_array(array, length);
} /* for element */
iteration++;
} /* while (this_iteration_had_a_swap) */
} /* bubble_sort */
What is the time complexity of bubble sort?
Suppose the array has length \( n \).
Well, the inner `for` loop always has \( n-1 \) iterations.
How many iterations does the outer `while` loop have?
Well, the outer loop will iterate until the whole array is sorted
(plus one extra iteration to determine that it is sorted).
On average, it’ll have to do that about \( \frac{1}{2} n \) times.
Therefore, the time complexity is \( O(n^2) \) – which is bad.
Selection Sort

Selection sort works by selecting the lowest (or highest) value in the list and swapping it with the first (or last) element, then finding the second lowest (or highest) and swapping it with the second (or next-to-last) element, and so on.

void selection_sort (int* array, int length)
{
    const int first_element = 0;
    int temporary;
    int element1, element2, minimum_element;
    void print_array(int* array, int length);
for (element1 = first_element;  
    element1 < (length - 1); element1++) {

    /* Find the minimum. */
    minimum_element = element1;
    for (element2 = element1 + 1;  
        element2 < length; element2++) {
        if (array[element2] < array[minimum_element]) {
            minimum_element = element2;
        } /* if (array[element2] < array[minimum_element]) */
        printf("Element %d is %d, minimum element at %d is %d.\n",  
               element2, array[element2],  
               minimum_element, array[minimum_element]);
    } /* for element2 */

    /* Swap! */
    temporary              = array[element1];
    array[element1]        = array[minimum_element];
    array[minimum_element] = temporary;

    printf("Minimum #%d had index %d",  
           element1 + 1, minimum_element);
    printf(" but now has index %d: \n", element1);
    print_array(array, length);
} /* for element1 */
} /* selection_sort */
I'm going to sort an array for you.
How many elements will the array have?
9
What are the 9 elements, unsorted?
-23  97  18  21   5 -86  64   0 -37
Selection Sort Run Minimum #1

-23  97  18  21   5  -86  64   0  -37
Element 1 is 97, minimum element at 0 is -23.
Element 2 is 18, minimum element at 0 is -23.
Element 3 is 21, minimum element at 0 is -23.
Element 4 is 5, minimum element at 0 is -23.
Element 5 is -86, minimum element at 5 is -86.
Element 6 is 64, minimum element at 5 is -86.
Element 7 is 0, minimum element at 5 is -86.
Element 8 is -37, minimum element at 5 is -86.
Minimum #1 had index 5 but now has index 0:
-86  97  18  21   5  -23  64   0  -37
Selection Sort Run Minimum #2

-86  97  18  21  5  -23  64  0  -37
Element 2 is 18, minimum element at 2 is 18.
Element 3 is 21, minimum element at 2 is 18.
Element 4 is 5, minimum element at 4 is 5.
Element 5 is -23, minimum element at 5 is -23.
Element 6 is 64, minimum element at 5 is -23.
Element 7 is 0, minimum element at 5 is -23.
Element 8 is -37, minimum element at 8 is -37.
Minimum #2 had index 8 but now has index 1:
-86  (-37)  18  21  5  -23  64  0  97
Selection Sort Run Minimum #3

-86 -37 18 21 5 -23 64 0 97
Element 3 is 21, minimum element at 2 is 18.
Element 4 is 5, minimum element at 4 is 5.
Element 5 is -23, minimum element at 5 is -23.
Element 6 is 64, minimum element at 5 is -23.
Element 7 is 0, minimum element at 5 is -23.
Element 8 is 97, minimum element at 5 is -23.
Minimum #3 had index 5 but now has index 2:
-86 -37 -23 21 5 18 64 0 97
Selection Sort Run Minimum #4

-86 -37 -23 21 5 18 64 0 97
Element 4 is 5, minimum element at 4 is 5.
Element 5 is 18, minimum element at 4 is 5.
Element 6 is 64, minimum element at 4 is 5.
Element 7 is 0, minimum element at 7 is 0.
Element 8 is 97, minimum element at 7 is 0.
Minimum #4 had index 7 but now has index 3:
-86 -37 -23 0 5 18 64 21 97
Selection Sort Run Minimum #5

-86 -37 -23  0  5  18  64  21  97
Element 5 is 18, minimum element at 4 is 5.
Element 6 is 64, minimum element at 4 is 5.
Element 7 is 21, minimum element at 4 is 5.
Element 8 is 97, minimum element at 4 is 5.
Minimum #5 had index 4 but now has index 4:
-86 -37 -23  0  5  18  64  21  97
Selection Sort Run Minimum #6

-86 -37 -23 0 5 18 64 21 97

Element 6 is 64, minimum element at 5 is 18.
Element 7 is 21, minimum element at 5 is 18.
Element 8 is 97, minimum element at 5 is 18.
Minimum #6 had index 5 but now has index 5:
-86 -37 -23 0 5 18 64 21 97
Selection Sort Run Minimum #7

-86  -37  -23  0   5   18   64   21   97
Element 7 is 21, minimum element at 7 is 21.
Element 8 is 97, minimum element at 7 is 21.
Minimum #7 had index 7 but now has index 6:
-86  -37  -23  0   5   18   21   64   97
Selection Sort Run Minimum #8

-86 -37 -23 0 5 18 21 64 97
Element 8 is 97, minimum element at 7 is 64.
Minimum #8 had index 7 but now has index 7:
-86 -37 -23 0 5 18 21 64 97
The 9 elements, sorted, are:
-86 -37 -23 0 5 18 21 64 97
Time Complexity of Selection Sort #1

for (element1 = first_element;
     element1 < (length - 1); element1++) {

    /* Find the minimum. */
    minimum_element = element1;
    for (element2 = element1 + 1;
         element2 < length; element2++) {
        if (array[element2] < array[minimum_element]) {
            minimum_element = element2;
        } /* if (array[element2] < array[minimum_element]) */
    } /* for element2 */

    /* Swap! */
    temporary = array[element1];
    array[element1] = array[minimum_element];
    array[minimum_element] = temporary;

    printf("Minimum #%d had index %d",
           element1, minimum_element);
    printf(" but now has Index %d:\n", element1);
    print_array(array, length);
} /* for element1 */
} /* selection_sort */
What is the time complexity of selection sort?

Suppose the array has length $n$.

Well, the outer $\texttt{for}$ loop always has $n-1$ iterations.

How many iterations does the inner $\texttt{for}$ loop have?

Well, for the first iteration of the outer loop, the inner loop has $n-1$ iterations; for the second iteration of the outer loop, the inner loop has $n-2$ iterations; and so on.

On average, the inner loop will have approximately $\frac{1}{2} n$ iterations.

Therefore, the time complexity is $\mathcal{O}(n^2)$ – which is bad.
Quick Sort #1

#include <stdio.h>
#include <stdlib.h>

void quicksort (int* array, int length)
{ /* quicksort */
    void quicksort_helper(int* array, int length,
                           int low, int high);

    quicksort_helper(array, length, 0, length - 1);
} /* quicksort */

http://www.inf.fh-flensburg.de/lang/algorithmen/sortieren/quick/quicken.htm
void quicksort_helper (int* array, int length, int low, int high)
{
    /* quicksort_helper */
    int temporary;
    int element1, element2, middle;
    int pivot;
    void print_array(int* array, int length);

    element1 = low;
    element2 = high;
    middle   = (low + high) / 2;
    pivot    = array[middle];
/* Partition this part of the array into halves. */
do {
    while (array[element1] < pivot) {
        element1++;
        printf("low %d, high %d, pivot %d, element1 %d\n", low, high, pivot, element1);
        print_array(array, length);
    } /* while (array[element1] < pivot) */
    while (array[element2] > pivot) {
        element2--;
        printf("low %d, high %d, pivot %d, element2 %d\n", low, high, pivot, element2);
        print_array(array, length);
    } /* while (array[element2] > pivot) */
/* Now, all values at indices between element1 and element2 are in the wrong place. */
if (element1 <= element2) {
  /* Swap! */
  temporary = array[element1];
  array[element1] = array[element2];
  array[element2] = temporary;
  element1++;
  element2--;
  printf("low %d, high %d, pivot %d",
         low, high, pivot);
  printf(" element1 %d, element2 %d:\n",
         element1, element2);
  print_array(array, length);
} /* if (element1 <= element2) */
} /* while (element1 <= element2); */
Quick Sort #5

/* Recursively sort the two halves. */
if (low < element2) {
    quicksort_helper(array, length, low, element2);
} /* if (low < element2) */
if (element1 < high) {
    quicksort_helper(array, length, element1, high);
} /* if (element1 < high) */
printf("After sorting, %d through %d is:\n", low, high);
    print_array(array, length);
} /* quicksort_helper */
Quick Sort Run Start

I'm going to sort an array for you.
How many elements will the array have?
9
What are the 9 elements, unsorted?
-23  97  18  21   5 -86  64   0 -37
Quick Sort Run Call: low 0, high 8

-23  97  18  21   5 -86  64   0 -37
low 0, high 8, pivot 5, element1 1

-23  97  18  21   5 -86  64   0 -37
low 0, high 8, pivot 5, element1 2, element2 7:

-23 -37  18  21   5 -86  64   0  97
low 0, high 8, pivot 5, element1 3, element2 6:

-23 -37   0  21   5 -86  64  18  97
low 0, high 8, pivot 5, element2 5

-23 -37   0  21   5 -86  64  18  97
low 0, high 8, pivot 5, element1 4, element2 4:

-23 -37   0 -86   5  21  64  18  97
low 0, high 8, pivot 5, element1 5, element2 3:
Quick Sort Run Call: low 0, high 3

-23 -37  0 -86| 5 21 64 18 97
low 0, high 3, pivot -37, element1 1, element2 2:
-86 -37  0 -23| 5 21 64 18 97
low 0, high 3, pivot -37, element2 1
-86 -37  0 -23| 5 21 64 18 97
low 0, high 3, pivot -37, element1 2, element2 0:
-86 -37  0 -23| 5 21 64 18 97
Quick Sort Run Call: low 2, high 3

-86 -37 [0] -23 | 5 21 64 18 97
low 2, high 3, pivot 0, element1 3, element2 2:
-86 -37 [-23] 0 | 5 21 64 18 97

After sorting, 2 through 3 is:
-86 -37 [-23] 0 | 5 21 64 18 97
Quick Sort Run Call: low 0, high 3 End

After sorting, 0 through 3 is:
-86 -37 -23 0 5 21 64 18 97
### Quick Sort Run Call: low 5, high 8

<table>
<thead>
<tr>
<th>-86</th>
<th>-37</th>
<th>-23</th>
<th>0</th>
<th>5</th>
<th>21</th>
<th>64</th>
<th>18</th>
<th>97</th>
</tr>
</thead>
<tbody>
<tr>
<td>low 5, high 8, pivot 64, element1 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-86</td>
<td>-37</td>
<td>-23</td>
<td>0</td>
<td>5</td>
<td>21</td>
<td>64</td>
<td>18</td>
<td>97</td>
</tr>
<tr>
<td>low 5, high 8, pivot 64, element2 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-86</td>
<td>-37</td>
<td>-23</td>
<td>0</td>
<td>5</td>
<td>21</td>
<td>64</td>
<td>18</td>
<td>97</td>
</tr>
<tr>
<td>low 5, high 8, pivot 64, element1 7, element2 6:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-86</td>
<td>-37</td>
<td>-23</td>
<td>0</td>
<td>5</td>
<td>21</td>
<td>18</td>
<td>64</td>
<td>97</td>
</tr>
</tbody>
</table>
Quick Sort Run Call: low 5, high 6

-86 -37 -23 0 5 | 21 | 18 | 64 97

low 5, high 6, pivot 21, element1 6, element2 5:
-86 -37 -23 0 5 | 18 | 21 | 64 97

After sorting, 5 through 6 is:
-86 -37 -23 0 5 | 18 21 | 64 97
Quick Sort Run Call: low 7, high 8

-86 -37 -23 0 5 18 21|64| 97
low 7, high 8, pivot 64, element2 7
-86 -37 -23 0 5 18 21|64| 97
low 7, high 8, pivot 64, element1 8, element2 6:
-86 -37 -23 0 5 18 21|64| 97

After sorting, 7 through 8 is:
-86 -37 -23 0 5 18 21|64| 97
Quick Sort Run Call: low 5, high 8 End

After sorting, 5 through 8 is:
-86 -37 -23 0 5 | 18 21 64 97 |
Quick Sort Run Call: low 0, high 8 End

After sorting, 0 through 8 is:
-86 -37 -23 0 5 18 21 64 97
Quick Sort Run End

The 9 elements, sorted, are:
-86 -37 -23 0 5 18 21 64 97
What is the time complexity of quick sort?
Suppose the array has length $n$.
Well, each call to `quicksort_helper` cuts the array in half.
But, each call to `quicksort_helper` can be accompanied by another call that covers the other half of the current subarray.
So, at each level of calls, most of the array gets visited.
So all we need to know is, how many levels of calls are there?
Time Complexity of Quick Sort #2

At each level of calls, the entire array gets visited. All we need to know is, how many levels of calls are there?

It turns out that, for an array of length $n$, there are $O(\log n)$ levels. So the total time complexity is $O(n \log n)$ – it’s quick!