Bit Representation Outline

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How Are Integers Represented in Memory?

In computers, all data are represented as contiguous sequences of bits.

An integer is represented as a sequence of 8, 16, 32 or 64 bits. For example:

\[ 97 = \begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & \\ \end{array} \]

What does this mean???
Decimal Number Representation (Base 10)

In the *decimal* number system (base 10), we have *10 digits*:

```
0 1 2 3 4 5 6 7 8 9
```

We refer to these as the *Arabic* digits. For details, see:

http://en.wikipedia.org/wiki/Arabic_numerals
Decimal (Base 10) Breakdown

\[
4721_{10} = 4000_{10} + 700_{10} + 20_{10} + 1_{10} =
\]

\[
4 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10^1 + 1 \cdot 10^0
\]

**Jargon:** \(4721_{10}\) is pronounced “four seven two one base 10,” or “four seven two one one decimal.”
Nonal Number Representation (Base 9)

In the *nonal* number system (base 9), we have 9 digits:

```
0 1 2 3 4 5 6 7 8
```

**NOTE**: No one uses nonal in real life; this is just an example.
Nonal (Base 9) Breakdown

\[ 4721_9 = \]
\[ 4000_9 + \]
\[ 700_9 + \]
\[ 20_9 + \]
\[ 1_9 = \]
\[ 4 \cdot 1000_9 + \]
\[ 7 \cdot 100_9 + \]
\[ 2 \cdot 10_9 + \]
\[ 1 \cdot 1_9 = \]
\[ 4 \cdot 9^3 + \]
\[ 7 \cdot 9^2 + \]
\[ 2 \cdot 9^1 + \]
\[ 1 \cdot 9^0 = \]

So: \[ 4721_9 = 3502_{10} \]

Jargon: \[ 4721_9 \] is pronounced “four seven two one base 9,” or “four seven two one nonal.”
Octal Number Representation (Base 8)

In the **octal** number system (base 8), we have 8 **digits**: 0 1 2 3 4 5 6 7

**NOTE**: Some computer scientists used to use octal in real life, but it has mostly fallen out of favor, because it’s been supplanted by base 16 (**hexadecimal**). Octal does show up a little bit in C character strings.
Octal (Base 8) Breakdown

\[
4721_8 = 4 \cdot 512_{10} + 7 \cdot 64_{10} + 2 \cdot 8_{10} + 1 \cdot 1_{10} = 2513_{10}
\]

So: \(4721_8 = 2513_{10}\)

Jargon: \(4721_8\) is pronounced “four seven two one base 8,” or “four seven two one octal.”
Trinary Number Representation (Base 3)

In the **trinary** number system (base 3), we have **3 digits**: 0 1 2

**NOTE**: No one uses trinary in real life; this is just an example.
Trinary (Base 3) Breakdown

2021<sub>3</sub> =

\[
\begin{align*}
2000_3 & + \\
0_3 & + \\
20_3 & + \\
1_3 & = \\
2 \cdot 1000_3 & + \\
0 \cdot 100_3 & + \\
2 \cdot 10_3 & + \\
1 \cdot 1_3 & = \\
2 \cdot 3^3 & + \\
0 \cdot 3^2 & + \\
2 \cdot 3^1 & + \\
1 \cdot 3^0 & = \\
\end{align*}
\]

\[2 \cdot 27_{10} + 0 \cdot 9_{10} + 2 \cdot 3_{10} + 1 \cdot 1_{10} = 61_{10}\]

So: 2021<sub>3</sub> = 61<sub>10</sub>

Jargon: 2021<sub>3</sub> is pronounced “two zero two one base 3,” or “two zero two one trinary.”
Binary Number Representation (Base 2)

In the **binary** number system (base 2), we have 2 **digits**:

```
    0 1
```

This is the number system that computers use internally.
# Binary (Base 2) Breakdown & Conversion

\[ 01100001_2 = \]

<table>
<thead>
<tr>
<th>Bit</th>
<th>Value</th>
<th>Calculation</th>
<th>Decimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2^7</td>
<td>0</td>
<td>0 \cdot 2^7</td>
<td>0 \cdot 128_{10}</td>
</tr>
<tr>
<td>2^6</td>
<td>1</td>
<td>1 \cdot 2^6</td>
<td>1 \cdot 64_{10}</td>
</tr>
<tr>
<td>2^5</td>
<td>1</td>
<td>1 \cdot 2^5</td>
<td>1 \cdot 32_{10}</td>
</tr>
<tr>
<td>2^4</td>
<td>0</td>
<td>0 \cdot 2^4</td>
<td>0 \cdot 16_{10}</td>
</tr>
<tr>
<td>2^3</td>
<td>1</td>
<td>1 \cdot 2^3</td>
<td>1 \cdot 8_{10}</td>
</tr>
<tr>
<td>2^2</td>
<td>0</td>
<td>0 \cdot 2^2</td>
<td>0 \cdot 4_{10}</td>
</tr>
<tr>
<td>2^1</td>
<td>0</td>
<td>0 \cdot 2^1</td>
<td>0 \cdot 2_{10}</td>
</tr>
<tr>
<td>2^0</td>
<td>1</td>
<td>1 \cdot 2^0</td>
<td>1 \cdot 1_{10}</td>
</tr>
</tbody>
</table>

\[ 97_{10} = \]

<table>
<thead>
<tr>
<th>Bit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2^7</td>
<td>0</td>
</tr>
<tr>
<td>2^6</td>
<td>1</td>
</tr>
<tr>
<td>2^5</td>
<td>1</td>
</tr>
<tr>
<td>2^4</td>
<td>0</td>
</tr>
<tr>
<td>2^3</td>
<td>0</td>
</tr>
<tr>
<td>2^2</td>
<td>0</td>
</tr>
<tr>
<td>2^1</td>
<td>0</td>
</tr>
<tr>
<td>2^0</td>
<td>1</td>
</tr>
</tbody>
</table>

**97_{10} = 01100001_2**
Counting in Decimal (Base 10)

In **base 10**, we **count** like so:

0,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

11, 12, 13, 14, 15, 16, 17, 18, 19, 20,

21, 22, 23, 24, 25, 26, 27, 28, 29, 30,

...

91, 92, 93, 94, 95, 96, 97, 98, 99, 100,

101, 102, 103, 104, 105, 106, 107, 108, 109, 110,

...

191, 192, 193, 194, 195, 196, 197, 198, 199, 200,

...

991, 992, 993, 994, 995, 996, 997, 998, 999, 1000,

...
Counting in Nonal (Base 9)

In **base 9**, we **count** like so:

0,

1, 2, 3, 4, 5, 6, 7, 8, 10,
11, 12, 13, 14, 15, 16, 17, 18, 20,
21, 22, 23, 24, 25, 26, 27, 28, 30,

... 81, 82, 83, 84, 85, 86, 87, 88, 100,
101, 102, 103, 104, 105, 106, 107, 108, 110,

... 181, 182, 183, 184, 185, 186, 187, 188, 200,

... 881, 882, 883, 884, 885, 886, 887, 888, 1000,

...
Counting in Octal (Base 8)

In **base 8**, we **count** like so:

0,
1, 2, 3, 4, 5, 6, 7, 10,
11, 12, 13, 14, 15, 16, 17, 20,
21, 22, 23, 24, 25, 26, 27, 30,
...
71, 72, 73, 74, 75, 76, 77, 100,
101, 102, 103, 104, 105, 106, 107, 110,
...
171, 172, 173, 174, 175, 176, 177, 200,
...
771, 772, 773, 774, 775, 776, 777, 1000,
...
Counting in Trinary (Base 3)

In **base 3**, we **count** like so:

0,
1, 2, 10,
11, 12, 20,
21, 22, 100,
101, 102, 110,
111, 112, 120,
121, 122, 200,
201, 202, 210,
211, 212, 220,
221, 222, 1000,
...

Counting in Binary (Base 2)

In **base 2**, we **count** like so:

0, 1,
10, 11,
100, 101, 110, 111,
1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111
10000, ...

Counting in Binary (Base 2) w/Leading 0s

In **base 2**, we sometimes like to put in *leading zeros*:

00000000, 00000001,
00000010, 00000011,
00000100, 00000101, 00000110, 00000111,
00001000, 00001001, 00001010, 00001011,
00001100, 00001101, 00001110, 00001111
00010000, ...
Counting in Binary Video

https://img-9gag-fun.9cache.com/photo/aq7Q4AZ_460svvp9.webm
## Adding Integers #1

<table>
<thead>
<tr>
<th></th>
<th>128</th>
<th>64</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2⁷</td>
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<td></td>
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<td></td>
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<td>2⁶</td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>2⁵</td>
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<tr>
<td>2⁴</td>
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<tr>
<td>2³</td>
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<td></td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2⁰</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
97_{10} = \begin{array}{cccccccc}
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\]

+ \[
15_{10} = \begin{array}{cccccccc}
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[
112_{10} = \begin{array}{cccccccc}
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\end{array}
\]
## Adding Integers #2

<table>
<thead>
<tr>
<th></th>
<th>128</th>
<th>64</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$97_{10}$ =</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$+ 06_{10}$ =</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$103_{10}$ =</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
% cat xadd.c
#include <stdio.h>

int main ()
{ /* main */
  int x;
  x = 97;
  printf("%d\n", x);
  x = x + 6;
  printf("%d\n", x);
  return 0;
} /* main */
% gcc -o xadd xadd.c
% xadd
  97
  103